

JOIM Conference

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Boston

ASSET ALLOCATION UNDER STRESS



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Introduction

Q4 2008 and Q1 2009 → What Should I Do? “All that ‘textbook’ Quant portfolio stuff they taught me was wrong”!!!!!!!

- There was no diversification across asset classes
- Everyone’s risk tolerance has gone down
- Get out of Equities into Bonds...Everyone!
(“It’s a market bottom when no-one wants to hold equities any more”)

$$\begin{aligned}\bar{S}_1 &= \frac{1}{n} \sum_{t=1}^n x_1^t \\ \bar{S}_2 &= \frac{1}{n} \sum_{t=1}^n x_2^t \\ HV_1^2 &= VAR(S_1) = \frac{1}{n-1} \sum_{t=1}^n (x_1^t - \bar{S}_1)^2 \\ HV_2^2 &= VAR(S_2) = \frac{1}{n-1} \sum_{t=1}^n (x_2^t - \bar{S}_2)^2 \\ cov(S_1, S_2) &= \frac{1}{n-1} \sum_{t=1}^n (x_1^t - \bar{S}_1)(x_2^t - \bar{S}_2) \\ corr(S_1, S_2) &= \frac{cov(S_1, S_2)}{\sqrt{VAR(S_1) * VAR(S_2)}} = \frac{cov(S_1, S_2)}{HV_1 * HV_2} \\ Beta(S_1, S_2) &= \frac{cov(S_1, S_2)}{VAR(S_2)} = corr(S_1, S_2) * \frac{HV_1}{HV_2}\end{aligned}$$



Did “Diversification Fail”?

Is Diversification No longer an Issue?

Consider simple example of crisis correlations tending to 1.0.

- 10 asset classes
- In “normal” times
 - » Volatility of each = 20%
 - » Correlation between each = 0.4
- In “crisis” times
 - » Volatility of each = 40%
 - » Correlation between is very high

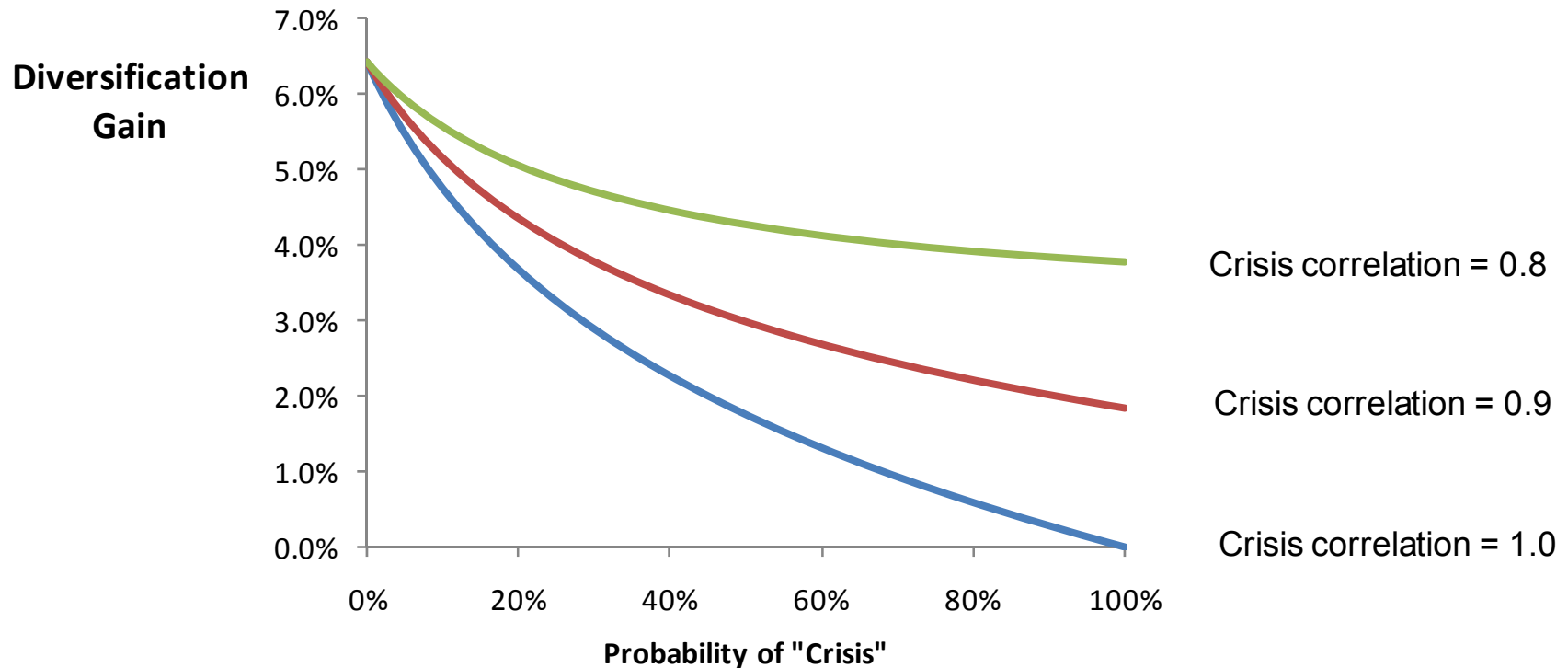


Did “Diversification Fail”?

Is Diversification No longer an Issue?

(contd.)

Probability of a "Crisis" and Diversification Gains



Revise Equity Return Projections Based on Technical Momentum Rule?

If cumulative return on S&P 500 over any five trading days is -10% or less:

- Exit market
- Return in 100 trading days

How much would you have saved in 2008?

- Exit date: October 7, 2008
- Avoided subsequent loss on S&P500 is 28.44%!



But...

Trigger Date (10% cumulative loss over five days)	100 Day Subsequent Return on the S&P500
5/28/1962	0.16%
10/19/1987	18.47%
8/31/1998	30.82%
4/14/2000	10.17%
9/21/2001	14.33%
7/22/2002	9.97%
Average	13.99%



Other Investors' Reactions....?

If think $\underline{E}(R)$ change: Consider other investors' actions: "Reflexivity" etc.

➤ Rational Investors

➤ "Irrational" Investors



Asset Allocation Response to the Crisis: Not everyone can get out at once: Market-Clearing Constraint

We will simplify by assuming only four asset classes:

- US Equity
- Developed (Foreign) Market Equity
- Emerging Market Equity
- Bonds

We will examine how optimal allocations should change in response to the crisis when we assume:

- Crisis significantly raises risk (volatility) of each asset class
- After the crisis asset class correlations become closer to 1.0
- After the crisis most investors' risk tolerance decreases since crisis makes everyone poorer



Before the Crisis:

We assume that before crisis average risk tolerance is 0.5 and riskless rate is 3%.

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.07%
Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.55%
Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
Bonds	40.00%	10.00%	0.40	0.35	0.30	1.00	4.72%

* Average risk tolerance = 0.5, riskless rate = 3%

Equilibrium returns are such that an investor with risk tolerance of 0.5 is willing to allocate 20% to US Equity, 22% to Developed Market Equity, 18% to Emerging Market Equity, and 40% to Bonds.



Optimal Allocations Before the Crisis

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.07%
Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.55%
Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
Bonds	40.00%	10.00%	0.40	0.35	0.30	1.00	4.72%

* Average risk tolerance = 0.5, riskless rate = 3%

Optimal Allocation for Given Risk Tolerance

	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 1.0$
US Equity	11.37%	13.53%	15.69%	17.84%	20.00%	22.16%	24.31%	26.47%	28.63%	25.28%
Dev Equity	9.99%	12.99%	15.99%	19.00%	22.00%	25.00%	28.01%	31.01%	34.01%	34.04%
Em Equity	0.18%	4.63%	9.09%	13.54%	18.00%	22.46%	26.91%	31.37%	35.82%	40.68%
Bonds	78.46%	68.85%	59.23%	49.62%	40.00%	30.38%	20.77%	11.15%	1.54%	0.00%



CRISIS HITS: Equities Markets Fall 40% and Bonds Fall 10%!

Allocation Before Crisis

	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 1.0$
US Equity	11.37%	13.53%	15.69%	17.84%	20.00%	22.16%	24.31%	26.47%	28.63%	25.28%
Dev Equity	9.99%	12.99%	15.99%	19.00%	22.00%	25.00%	28.01%	31.01%	34.01%	34.04%
Em Equity	0.18%	4.63%	9.09%	13.54%	18.00%	22.46%	26.91%	31.37%	35.82%	40.68%
Bonds	78.46%	68.85%	59.23%	49.62%	40.00%	30.38%	20.77%	11.15%	1.54%	0.00%

Allocation after Equities Fall 40% and Bonds Fall 10%

	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 1.0$
US Equity	8.17%	10.06%	12.10%	14.30%	16.67%	19.23%	22.03%	25.07%	28.41%	25.28%
Dev Equity	7.17%	9.66%	12.34%	15.22%	18.33%	21.71%	25.37%	29.37%	33.75%	34.04%
Em Equity	0.13%	3.45%	7.01%	10.85%	15.00%	19.49%	24.38%	29.71%	35.55%	40.68%
Bonds	84.53%	76.83%	68.55%	59.63%	50.00%	39.57%	28.22%	15.84%	2.29%	0.00%



Investment Environment After Crisis

Before

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.07%
Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.55%
Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
Bonds	40.00%	10.00%	0.40	0.35	0.30	1.00	4.72%

* Average risk tolerance = 0.5, riskless rate = 3%

We assume that:

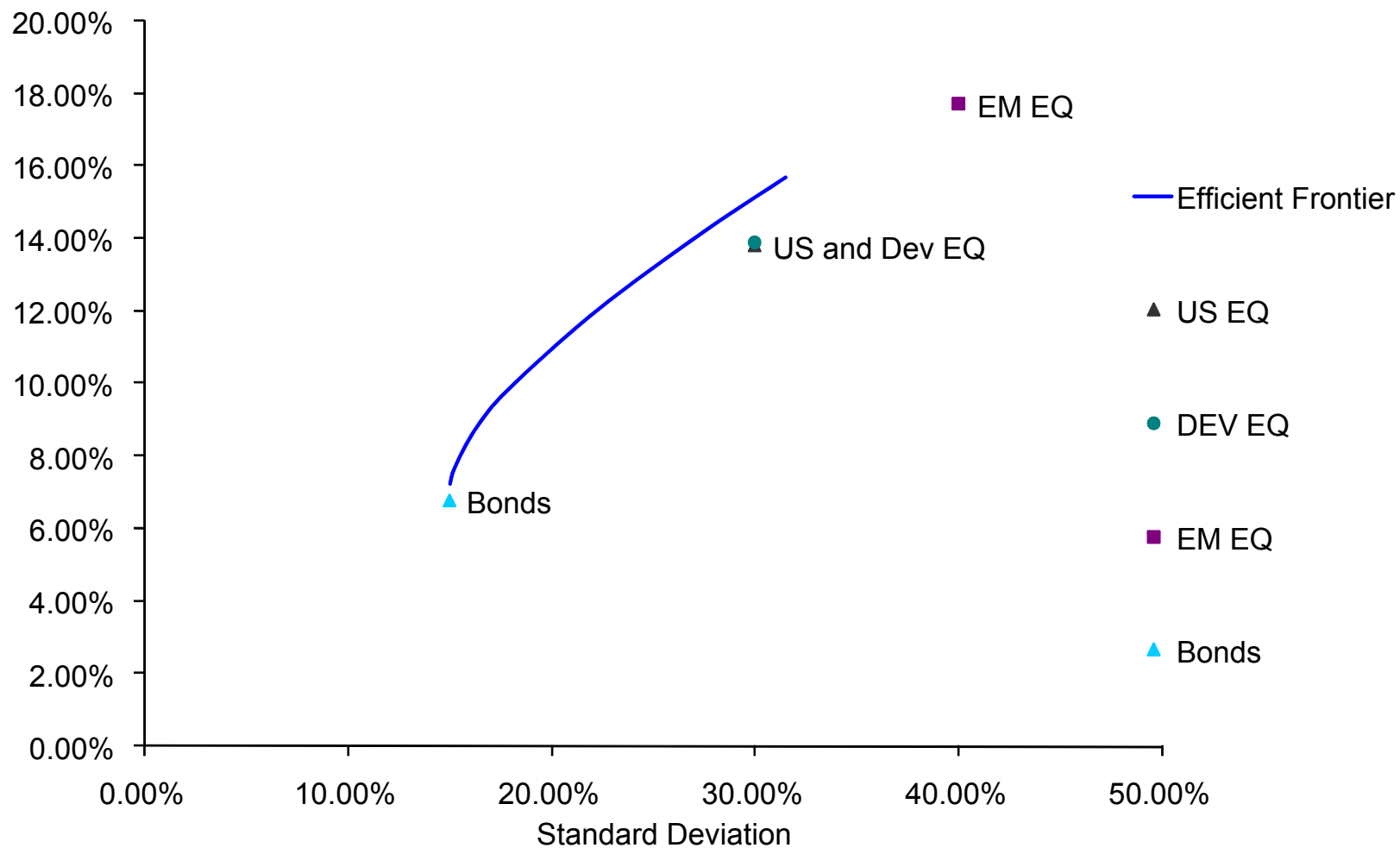
- Everyone's risk tol. falls by 0.10.
- Asset volatilities increase.
- Correlations increase.
- Riskless rate falls to 1%.

After

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	16.67%	30.00%	1.00	0.75	0.70	0.50	13.81%
Dev Equity	18.33%	30.00%	0.75	1.00	0.70	0.50	13.90%
Em Equity	15.00%	40.00%	0.70	0.70	1.00	0.45	17.73%
Bonds	50.00%	15.00%	0.50	0.50	0.45	1.00	6.79%

* Average risk tolerance = 0.4, riskless rate = 1%

Efficient Frontier After the Crisis



How much change in allocations?

Allocation after Equities Fall 40% and Bonds Fall 10%

	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 1.0$
US Equity	8.17%	10.06%	12.10%	14.30%	16.67%	19.23%	22.03%	25.07%	28.41%	25.28%
Dev Equity	7.17%	9.66%	12.34%	15.22%	18.33%	21.71%	25.37%	29.37%	33.75%	34.04%
Em Equity	0.13%	3.45%	7.01%	10.85%	15.00%	19.49%	24.38%	29.71%	35.55%	40.68%
Bonds	84.53%	76.83%	68.55%	59.63%	50.00%	39.57%	28.22%	15.84%	2.29%	0.00%

Post Crisis Optimal Allocation for Given Risk Tolerance (note shift in risk tolerances)

	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
US Equity	0.61%	6.07%	9.94%	13.31%	16.67%	20.03%	23.39%	26.75%	30.11%	27.21%
Dev Equity	0.65%	6.48%	10.78%	14.56%	18.33%	22.11%	25.89%	29.67%	33.44%	30.96%
Em Equity	0.00%	0.00%	4.32%	9.66%	15.00%	20.34%	25.68%	31.02%	36.36%	41.83%
Bonds	98.75%	87.45%	74.96%	62.48%	50.00%	37.52%	25.04%	12.56%	0.08%	0.00%

Trading

	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
US Equity	-7.56%	-4.00%	-2.16%	-0.99%	0.00%	0.79%	1.36%	1.68%	1.70%	1.93%
Dev Equity	-6.53%	-3.18%	-1.56%	-0.67%	0.00%	0.41%	0.52%	0.30%	-0.31%	-3.08%
Em Equity	-0.13%	-3.45%	-2.69%	-1.19%	0.00%	0.85%	1.30%	1.31%	0.81%	1.15%
Bonds	14.21%	10.62%	6.41%	2.85%	0.00%	-2.04%	-3.18%	-3.28%	-2.20%	0.00%



How much change in allocations? (contd.)

Given the assumptions we have made, those more risk tolerant than average will tend to modestly increase their allocations to riskier assets, while those less risk tolerant than average will tend to decrease.

	Trading									
	$\rho = 0.01$	$\rho = 0.1$	$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.4$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
US Equity	-7.56%	-4.00%	-2.16%	-0.99%	0.00%	0.79%	1.36%	1.68%	1.70%	1.93%
Dev Equity	-6.53%	-3.18%	-1.56%	-0.67%	0.00%	0.41%	0.52%	0.30%	-0.31%	-3.08%
Em Equity	-0.13%	-3.45%	-2.69%	-1.19%	0.00%	0.85%	1.30%	1.31%	0.81%	1.15%
Bonds	14.21%	10.62%	6.41%	2.85%	0.00%	-2.04%	-3.18%	-3.28%	-2.20%	0.00%

Prices (and asset returns) adjust until demand of those wishing to buy just equals supply of those wishing to sell.



What if correlations were even higher?

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	16.67%	30.00%	1.00	0.90	0.90	0.50	15.33%
Dev Equity	18.33%	30.00%	0.90	1.00	0.90	0.50	15.36%
Em Equity	15.00%	40.00%	0.90	0.90	1.00	0.45	19.83%
Bonds	50.00%	15.00%	0.50	0.50	0.45	1.00	6.79%

* Average risk tolerance = 0.4, riskless rate = 1%



Allocation and Trading

Allocation after Equities Fall 40% and Bonds Fall 10%

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
US Equity	8.17%	10.06%	12.10%	14.30%	16.67%	19.23%	22.03%	25.07%	28.41%	25.28%
Dev Equity	7.17%	9.66%	12.34%	15.22%	18.33%	21.71%	25.37%	29.37%	33.75%	34.04%
Em Equity	0.13%	3.45%	7.01%	10.85%	15.00%	19.49%	24.38%	29.71%	35.55%	40.68%
Bonds	84.53%	76.83%	68.55%	59.63%	50.00%	39.57%	28.22%	15.84%	2.29%	0.00%

Post Crisis Optimal Allocation for Given Risk Tolerance (note shift in risk tolerances)

	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
US Equity	0.66%	6.58%	13.15%	15.47%	16.67%	17.86%	19.06%	20.26%	21.46%	17.90%
Dev Equity	0.70%	6.99%	13.99%	16.72%	18.33%	19.95%	21.56%	23.18%	24.79%	21.65%
Em Equity	0.00%	0.00%	0.00%	6.63%	15.00%	23.37%	31.74%	40.11%	48.48%	60.44%
Bonds	98.64%	86.43%	72.86%	61.18%	50.00%	38.82%	27.64%	16.46%	5.28%	0.00%

Trading

	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
US Equity	-7.51%	-3.49%	1.05%	1.17%	0.00%	-1.37%	-2.97%	-4.81%	-6.95%	-7.38%
Dev Equity	-6.47%	-2.67%	1.65%	1.50%	0.00%	-1.76%	-3.81%	-6.20%	-8.96%	-12.39%
Em Equity	-0.13%	-3.45%	-7.01%	-4.22%	0.00%	3.87%	7.36%	10.39%	12.92%	19.77%
Bonds	14.11%	9.60%	4.31%	1.55%	0.00%	-0.75%	-0.58%	0.61%	2.99%	0.00%

“Flight to Quality”

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.07%
Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.55%
Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
Bonds	40.00%	10.00%	0.40	0.35	0.30	1.00	4.72%

* Average risk tolerance = 0.5, riskless rate = 3%

	Market Weights	Standard Deviation	Correlations				Equilibrium Return*
			US Equity	Dev Equity	Em Equity	Bonds	
US Equity	16.67%	30.00%	1.00	0.90	0.90	-0.10	11.95%
Dev Equity	18.33%	30.00%	0.90	1.00	0.90	-0.10	11.99%
Em Equity	15.00%	40.00%	0.90	0.90	1.00	0.00	16.45%
Bonds	50.00%	15.00%	-0.10	-0.10	0.00	1.00	3.42%

* Average risk tolerance = 0.4, riskless rate = 1%



“Flight to Quality”: Allocation and Trading

Allocation after Equities Fall 40% and Bonds Fall 10%

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
US Equity	8.17%	10.06%	12.10%	14.30%	16.67%	19.23%	22.03%	25.07%	28.41%	25.28%
Dev Equity	7.17%	9.66%	12.34%	15.22%	18.33%	21.71%	25.37%	29.37%	33.75%	34.04%
Em Equity	0.13%	3.45%	7.01%	10.85%	15.00%	19.49%	24.38%	29.71%	35.55%	40.68%
Bonds	84.53%	76.83%	68.55%	59.63%	50.00%	39.57%	28.22%	15.84%	2.29%	0.00%

Post Crisis Optimal Allocation for Given Risk Tolerance (note shift in risk tolerances)

	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
US Equity	11.88%	14.98%	18.43%	20.05%	16.67%	13.28%	9.89%	6.50%	12.20%	11.08%
Dev Equity	11.92%	15.40%	19.26%	21.30%	18.33%	15.36%	12.39%	9.42%	15.53%	14.83%
Em Equity	0.00%	0.00%	0.00%	3.16%	15.00%	26.84%	38.69%	50.53%	55.49%	65.61%
Bonds	76.19%	69.62%	62.31%	55.49%	50.00%	44.51%	39.03%	33.54%	16.78%	8.47%

Trading

	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
US Equity	3.72%	4.92%	6.33%	5.76%	0.00%	-5.96%	-12.13%	-18.57%	-16.21%	-14.20%
Dev Equity	4.75%	5.74%	6.92%	6.08%	0.00%	-6.34%	-12.98%	-19.95%	-18.22%	-19.21%
Em Equity	-0.13%	-3.45%	-7.01%	-7.70%	0.00%	7.35%	14.31%	20.82%	19.94%	24.93%
Bonds	-8.34%	-7.21%	-6.24%	-4.15%	0.00%	4.95%	10.81%	17.70%	14.49%	8.47%



Next Steps

- ❑ **Across Investors: Different Risk Tolerances => Different pre-Crisis Allocations => Different Wealth Effect => Post-Crisis Event => Market Risk Appetite has changed**
- ❑ **Predictable Regressivity in Parameters => “Macro Bets”**
- ❑ **If know allocation and characteristics of other investors, use that information in the “market-clearing” constraint: Component of a model of liquidity in crises? Separate into rational (infer risk tolerance from holdings) and others**
- ❑ **Overlap with “Who Should Buy Insurance” (and how much)**



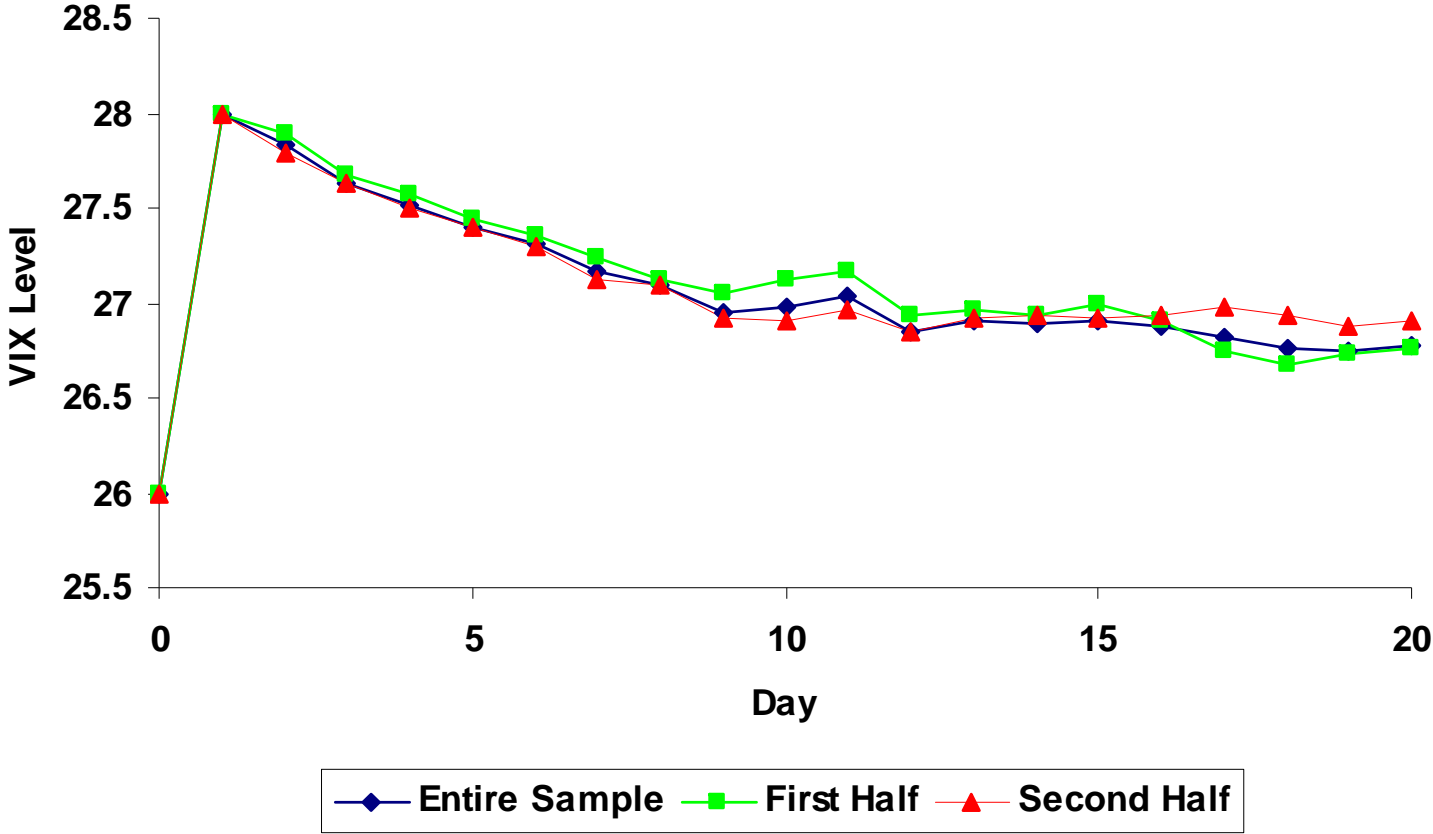
Predictable Regressivity in Parameters?

- **Assumptions about Expected Returns**
 - **Examples during Crisis**
 - **Observationally equivalent to differences in risk tolerances and differences in liability structure?**
- **Shifts Through Time in Volatility and Factor Structure => Quantal model adapts to shifts => “Fat Tails” in unconditional returns => Objective function for unconditional distribution?**



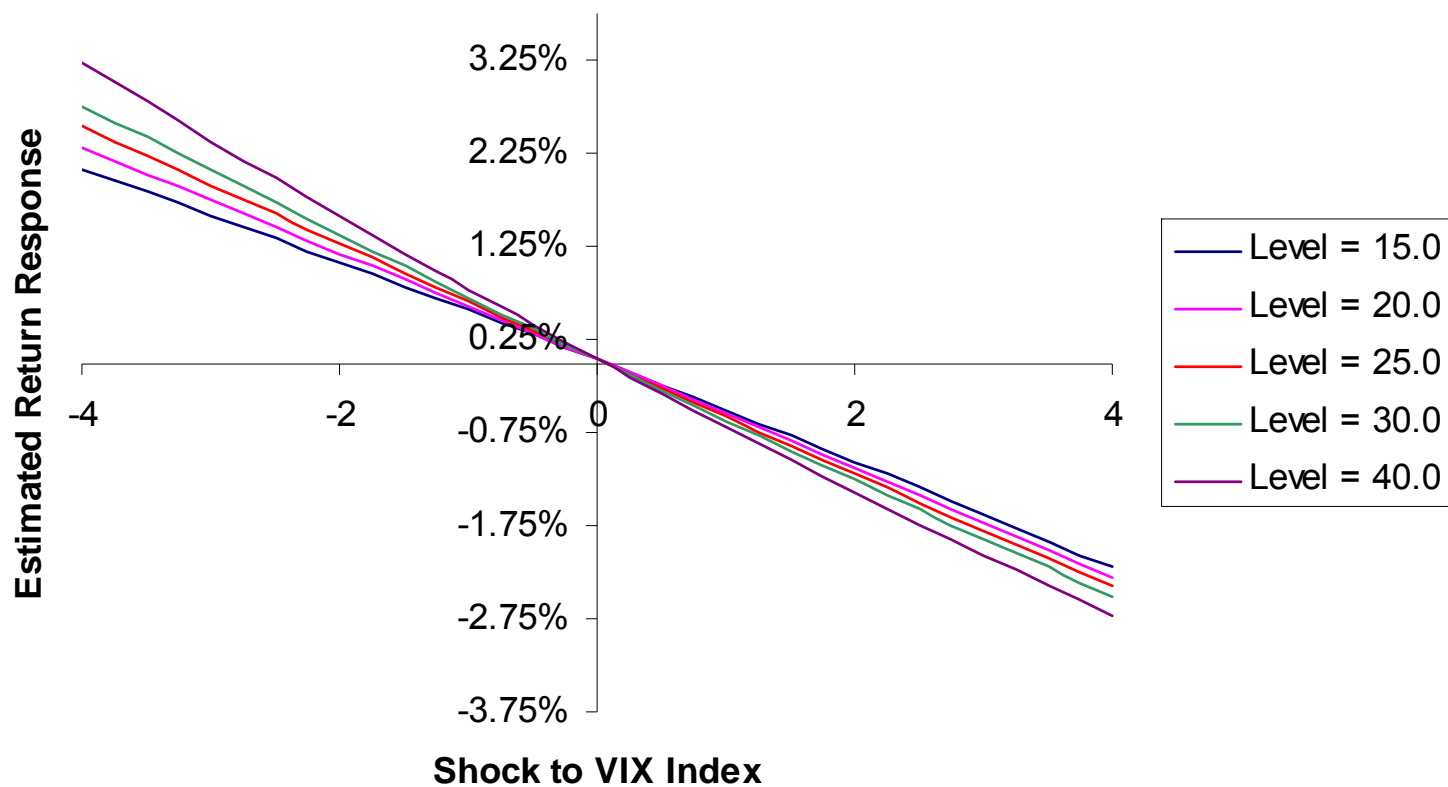
Shocks to VIX and Regressivity

Impulse Response: Day 1 Shock to Vix of +2.0



Shocks to VIX and Contemporaneous Returns

Estimated Contemporaneous Return Response to VIX Shock



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